



# Reaction to Technology Shocks in Markov-switching Structural VARs: Identification via Heteroskedasticity

Aleksei Netšunajev

Working Paper Series

**6/2012**

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ISBN 978-9949-493-09-8  
Eesti Pank. Working Paper Series, ISSN 1406-7161; 6

# Reaction to Technology Shocks in Markov-switching Structural VARs: Identification via Heteroskedasticity

Aleksei Netsunajev\*

## Abstract

The paper reconsiders the conflicting results in the debate connected to the effects of technology shocks on hours worked. Given the major dissatisfaction with the just-identifying long-run restrictions, I analyze whether the restrictions used in the literature are consistent with the data. Modeling volatility of shocks using Markov switching structure allows to obtain additional identifying information and perform tests of the restrictions that were just-identifying in classical structural vector autoregressive analysis. Using six ways of identifying technology shocks, I find that not all of them are supported by the data. There is no clear-cut evidence in favor of a positive reaction of hours to technology shocks.

JEL Code: C32

Keywords: technology shocks, Markov switching model, heteroskedasticity

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The views expressed are those of the author and do not necessarily represent the official views of Eesti Pank.

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\*The study received Eesti Pank's 2012 research award.

The author would like to thank Professor Helmut Lütkepohl, participants in the seminar at the Bank of Estonia and the European University Institute, as well as an anonymous referee for useful comments on an earlier version of the paper and Professor Valery Ramey for providing the data. A more detailed version of this paper, containing Appendix A, is available upon request.

## Non-technical summary

A standard real business cycle model implies that per capita hours worked rise after a permanent shock to technology. This prediction is the focus of literature that assesses whether or not it is consistent with the data. The general conclusion reached is that it is not. Not surprisingly this result has attracted a lot of attention as technology shocks are a significant source of fluctuations in productivity and employment.

In the literature a variety of methods are used to study the question, but the most common is based on structural vector autoregressive (VAR) models. In the paper six different previously known identification schemes of technology shocks are discussed in the context of structural VARs. The most common identifying assumption implies that only technology shocks have long run effects on labour productivity. Other studies propose analysing non-permanent technology shocks, permanent real wage shocks and permanent TFP shocks, or controlling for the effects of capital tax and disentangling investment-specific and neutral technology shocks. In the conventional framework, potentially competing restrictions are just identified and hence they are not testable. In contrast, the present setup of the econometric model allows for the extraction of additional information out of the data and for testing and not just identifying long run restrictions.

Thus the aim of the current paper is to reconsider the reaction of hours worked to technology shocks and to relax some of the assumptions common in this literature. For that purpose a series of Markov-switching (MS) models are estimated that allow changes in volatility and intercept to be captured, providing a framework for testing for the validity of the identifying restrictions and for assessing the labelling of the identified shocks as technology shocks.

In the model setup it is assumed that the time dependent intercept and the distribution of the reduced form error term depend on a discrete Markov process. The changes in the volatility of the residuals are used in this framework to test whether the identified shocks are in line with the properties of the data.

For the purpose of validating restrictions, MS models with two and three states are estimated. The outcome of the testing can be briefly summarised as follows:

1. The identification of permanent technology shocks, non-permanent technology shocks and permanent real wage shocks is supported for the models with two and three Markov states.
2. Permanent TFP shocks and permanent technology shocks after controlling for capital tax are supported in two-state models, but fail to comply with the state invariant instantaneous effects of shocks in three-state models.

3. Disentanglement of investment specific and neutral technology shocks is not supported by the data independently of the number of states. However, a neutral technology shock can be identified in the system.

Given that the majority of the identification schemes were supported by the data, an impulse response analysis may be performed for supported identification. The variety of impulse responses studied does not provide clear-cut strong evidence in favour of a positive and significant reaction of hours to different technology shocks, although it is plausible for some models.

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# 1. Introduction

A standard real business cycle model implies that per capita hours worked rise after a permanent shock to technology. This prediction is at the center of the literature that assesses whether it is consistent with the data. The general conclusion reached is that it is not. Not surprisingly, the result has attracted a lot of attention as technology shocks are a significant source of fluctuations in productivity and employment.

In the literature one can find a variety of methods used to study reaction of hours worked to technology shocks, but the most common is based on structural vector autoregressive (SVAR) models. In a seminal paper, Gali (1999) identifies the technology shocks or, put differently, permanent productivity shocks, using long-run restrictions and he finds that hours worked fall after a positive technology shock. Several papers consider similar systems as in Gali (1999) and try to assess the validity of the identifying restrictions. A similar identification is used in Gali et al. (2003), Christiano et al. (2003), Francis and Ramey (2005), and Francis and Ramey (2009). The study by Francis and Ramey (2005) questions whether the shocks that are identified as in Gali (1999) can be classified as technology shocks. Using different identifying assumptions, they find that all but one specification produced the result similar to Gali (1999). In other words, Francis and Ramey (2005) show that permanent real wage and permanent productivity shocks after controlling for capital tax rate produce a negative reaction of hours worked.

Christiano et al. (2003) find that treating per capita hours worked as a difference stationary process yields the result that hours worked fall after the technology shock; if, on the contrary, hours worked are assumed to be a stationary process, the result is opposite: hours worked rise after the technology shock. Fernald (2007) and Francis and Ramey (2009) argue that there are low frequency movements in hours per capita that may distort the results of the SVAR in Christiano et al. (2003). After either detrending the data (Fernald (2007)) or applying a filter to the data (Francis and Ramey (2009)), the response of hours worked to a neutral technology shock becomes negative.

Fisher (2002) proposes to disentangle investment specific and neutral technology shocks. Similarly, Canova et al. (2010) consider the effects of neutral and investment-specific technology shocks on hours. Both studies show that hours worked fall in response to neutral shocks and increase in response to investment-specific shocks. Chang and Hong (2006) propose to identify the permanent total factor productivity (TFP) shocks in a way that is similar to Gali (1999). They show that the reaction of hours worked to a permanent TFP shock is positive.

It should be noted that the studies listed above may share some common shortcomings. First, the underlying assumptions just-identify the macroeconomic shocks and leave no place for the data to speak out against the restrictions. The problem of just-identified shocks is discussed, among others, by Lanne and Lütkepohl (2008), Lanne et al. (2010), and Herwartz and Lütkepohl. Second, studies of technology shocks (for example, Gali (1999), Francis and Ramey (2005), Christiano et al. (2003), Canova et al. (2010), Chang and Hong (2006)) ignore relevant features of the data, namely heteroskedasticity. The presence of time-varying volatility is extensively discussed and documented by Kim and Nelson (1999), McConnell and Perez-Quiros (2000), Blanchard and Simon (2001), Stock and Watson (2003), so it should be taken into account.

It is useful to take into account heteroskedasticity as it allows additional identifying information to be extracted from the data (Rigobon (2003)). In the present context this is important given the mixed evidence on the reaction of hours on technology shocks. Modeling heteroskedasticity can be used as a way of validating the restrictions that are just-identifying in a conventional SVAR analysis and for checking how different identification methods comply with the properties of the data.

Thus the aim of the current paper is to reconsider the reaction of hours worked to technology shocks and to relax some of the assumptions common in this literature. For this purpose, I estimate a series of Markov-switching (MS) models that allow the changes in volatility and intercept to be captured, provide a framework to test for the validity of the identifying restrictions, and assess the labeling of identified shocks as technology shocks. The model used in the paper is a modified version of the model used by Lanne et al. (2010) and Herwartz and Lütkepohl (2011).

The rest of the paper is organized as follows. I provide additional motivations for the paper, while different identification schemes of technology shocks and the data are discussed in Section 2. In Section 3 the structural MS-VAR model deployed in the current analysis is described. Section 4 provides the empirical analysis. The last section concludes.

## 2. Identification of shocks

Consider a standard  $K$ -dimensional reduced form VAR with  $p$  lags:

$$Y_t = \nu + A_1 Y_{t-1} + \dots + A_p Y_{t-p} + U_t, \quad (1)$$

where  $\nu$  is a constant intercept term, the  $A_j$ s ( $j = 1, \dots, p$ ) are  $(K \times K)$  coefficient matrices and  $U_t$  is a zero-mean error term.

In a conventional SVAR model, the structural shocks are usually obtained from the reduced form residuals by a linear transformation,  $\varepsilon_t = B^{-1}U_t$  or  $B\varepsilon_t = U_t$ , where  $B$  is such that  $\varepsilon_t$  has identity covariance matrix, that is,  $\varepsilon_t \sim (0, I_K)$ , and the reduced form residual covariance matrix is decomposed as  $E(U_t U_t') = \Sigma_U = BB'$ . To obtain unique structural shocks, one needs to place  $K(K-1)/2$  restrictions. For this reason the  $B$  matrix is often assumed to be lower triangular. Thus the  $B$  is the matrix of instantaneous effects of the unique structural shocks.

In the related technology shock literature, a bivariate system is usually considered in the spirit of Gali (1999). Using long run restrictions, one identifies two kinds of shocks: technology shocks and non-technology shocks. The shocks are identified in the following system, which is a moving average representation of a VAR:

$$\begin{bmatrix} \Delta x_t \\ \Delta n_t \end{bmatrix} = \begin{bmatrix} C_{11}(L) & C_{12}(L) \\ C_{21}(L) & C_{22}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_t^z \\ \varepsilon_t^m \end{bmatrix} \quad (2)$$

where  $x_t$  denotes the log of labor productivity,  $n_t$  denotes the log of labor input,  $\varepsilon_t^z$  is the technology shock and  $\varepsilon_t^m$  is the non-technology shock,  $C_{ij}(L)$  is a polynomial in the lag operator and  $\Delta$  is the difference operator.

In the present paper I follow the strategy proposed by Blanchard and Quah (1989) and place the restrictions on the total impact matrix  $\Xi_\infty = (I_K - A_1 - \dots - A_p)^{-1}B$ , which is identical to restricting the system in (2). It should be noted that the restrictions on  $\Xi_\infty$  can be transformed to the restrictions on  $B$  as shown in Lütkepohl (2005).

The most common identifying assumption restricts  $C_{12}(1) = 0$ , implying that only technology shocks have long-run effects on labor productivity (Gali (1999)). The non-technology shocks could thus be interpreted as demand shocks (Gali (1999)).

Another way of identifying technology shocks in the bivariate system is proposed by Francis and Ramey (2005). They argue that technology shocks should not have a long-run effect on hours or, put differently, they exclude permanent technology shocks. This restriction is implemented by constraining  $C_{21}(1) = 0$  above. Francis and Ramey (2005) argue that the resulting residuals in the productivity equation may contain other shocks in addition to the productivity shock. For instance, these could be monetary shocks that have no long-run effect on hours. Therefore this identification is different from the original one in Gali (1999) and may be problematic.

Francis and Ramey (2005) consider an alternative long-run restriction involving real wages using the theoretical result, i.e. that only a technology

shock should have a permanent effect on real wages. Thus, an alternative way to identify the technology shock is to substitute real wages for productivity and to impose  $C_{12}(1) = 0$ .

Francis and Ramey (2005) discuss the notion that permanent changes in capital income taxation can also have permanent effects on productivity. To control for this, they include current and four lags of the level of capital tax rates as exogenous variables in the VAR. On the contrary, I add the capital income tax series to the system above and untangle the technology shocks and capital income tax shocks using the long-run restrictions provided in Francis and Ramey (2005). Both technology shocks and capital income tax shocks can affect labor productivity in the long run, while a non-technology shock cannot. Further, permanent shifts in technology should not affect the long-run labor supply, while a capital income tax shock can have permanent effects on labor. Note that the described system would not be identified in the conventional SVAR, while the restrictions are testable in the MS-VAR framework.

Following Chang and Hong (2006), one can identify the permanent TFP shocks for the aggregate economy. This is done by substituting a TFP measure for productivity and imposing  $C_{12}(1) = 0$ .

Further, augmenting the bivariate system with the price of investment, one can disentangle investment-specific technology shocks and neutral technology shocks. Solely investment-specific technology shocks affect the price of investment in the long run, while both investment-specific and neutral shocks affect labor productivity in the long run. The identification corresponds to a lower-triangular  $\Xi_\infty$  matrix for the ordering of variables price of investment, productivity and hours.

Table 1 summarizes the variations of technology shocks used in the subsequent analysis. Notation for the variables is as follows:  $x_t$  log of labor productivity,  $n_t$  log of per capita hours worked,  $w_t$  log of real wage,  $TFP_t$  measure of total factor productivity,  $\tau_t$  measure of capital tax,  $i_t$  log of real price of investment.

Table 1: Models used to study technology shocks

	Used by	Data	Restrictions
Model 1	Gali (1999)	$y_t = [\Delta x_t, \Delta n_t]'$	$C_{12}(1) = 0$
Model 2	Francis and Ramey (2005)	$y_t = [\Delta x_t, \Delta n_t]'$	$C_{21}(1) = 0$
Model 3	Francis and Ramey (2005)	$y_t = [\Delta w_t, \Delta n_t]'$	$C_{12}(1) = 0$
Model 4	Chang and Hong (2006)	$y_t = [\Delta TFP_t, \Delta n_t]'$	$C_{12}(1) = 0$
Model 5	Francis and Ramey (2005)	$y_t = [\Delta x_t, \Delta n_t, \tau_t]'$	$C_{12}(1) = C_{21}(1) = 0$
Model 6	Canova et al. (2010)	$y_t = [\Delta i_t, \Delta x_t, \Delta n_t]'$	$C(1)$ lower triangular

I use quarterly data from 1947:Q1 through 2010:Q4 to estimate the models for permanent technology and non-permanent technology shocks. The data is obtained from the Bureau of Labor Statistics. Standard ADF tests for both productivity and hours indicate the presence of a unit root. Christiano et al. (2003) argue that hours per capita cannot logically have a unit root as it is a bounded process. However, hours series have low frequency movements that should be taken into account (Fernald (2007), Francis and Ramey (2009)). In the present paper I use hours series in first differences, as it is consistent with the technology shock literature. I use the data kindly provided by Professor Valery Ramey to estimate models for permanent real wage shocks and technology shocks after controlling for capital income tax. The data runs from 1947Q1 through 2003Q1 for the real wage model and through 1997Q4 for capital income tax model. Further, I use publicly available data from Chang and Hong (2006) and Canova et al. (2010) for permanent TFP (yearly data) and investment specific technology shock models. All variables except the tax rate are entered in logarithms. Lag order four is selected for all the datasets consistently with the previous studies. One exception is the permanent TFP data where lag order two, as suggested by the Akaike information criterion (AIC), has been chosen.

### **3. The Model**

#### **3.1. Markov Switching SVAR**

Identification via heteroskedasticity initially appeared with Rigobon (2003). In SVAR analysis, it is proposed and used by Rigobon and Sack (2003) and Lanne and Lütkepohl (2008), among others. These authors show that if there are exogenously generated changes in the volatility of the shocks, the structural parameters could be effectively recovered from the reduced form model. This identification is based on the assumptions that the system is stable over time (the effects of shocks are the same regardless of the volatility regime) and that the structural shocks are orthogonal. These assumptions are usually implicit in the conventional structural VAR analysis, and hence are no more restrictive than usual. In particular, they are also common to the technology shock literature.

In the present paper I consider conditional heteroskedasticity, which allows for changes in the volatility to be determined from the data. I use the approach proposed by Lanne et al. (2010) and model the changes in volatility and intercept by a Markov regime-switching (MS) mechanism. It should be noted that the approach does not label shocks economically but is rather a tool to

test whether economic restrictions that are just-identifying in the conventional SVAR are consistent with the data. Specifically, I consider a modified version of the model by Lanne et al. (2010).

Consider the VAR( $p$ ):

$$Y_t = \nu_{s_t} + A_1 Y_{t-1} + \cdots + A_p Y_{t-p} + U_t, \quad (3)$$

I assume that the time dependent intercept  $\nu_{s_t}$  as well as the distribution of the reduced form error term  $U_t$  depend on a discrete Markov process  $s_t$  ( $t = 0, \pm 1, \pm 2, \dots$ ) with states  $1, \dots, M$  and transition probabilities

$$p_{ij} = \Pr(s_t = j | s_{t-1} = i), \quad i, j = 1, \dots, M.$$

The conditional distribution of  $U_t$  given  $s_t$  is assumed to be normal,

$$U_t | s_t \sim N(0, \Sigma_{s_t}). \quad (4)$$

In addition to the state dependent covariance matrices, I also allow the intercept term  $\nu_{s_t}$  to be dependent on the Markov process. Models with similar features, changes in covariances and intercept, are used in the empirical business cycle literature as, for example, in Hamilton (1989) and Krolzig (1997). Fernald (2007), using the data similar to the data I use, tests for structural breaks in the productivity growth series and finds them to be likely. Potential breaks in the intercept of the hours series are also discussed by Canova et al. (2010). Therefore, the model deployed in the subsequent analysis must also capture potential non-regularities in the intercept. In the following I will stick to the notation similar to Krolzig (1997). MSIH( $M$ )-VAR( $p$ ) will denote models with changes in the intercept and volatility where  $M$  denotes the number of Markov states and  $p$  the lag length.

The changes in the volatility of the residuals are used in this framework to test whether the identified shocks are in line with the properties of the data. For instance, if there are two volatility states ( $M = 2$ ), then a decomposition of the covariance matrices  $\Sigma_1 = BB'$  and  $\Sigma_2 = B\Lambda_2 B'$  exists, where  $\Lambda_2 = \text{diag}(\lambda_{21}, \dots, \lambda_{2K})$  is a diagonal matrix with positive diagonal entries. The  $\Lambda_2$  matrix is thus the matrix of relative variances. Suppose  $\lambda_{2i}$ s are all distinct. Then the decomposition is unique up to changes in the sign and permutations of the columns of  $B$  and corresponding changes in the ordering of the weighting matrix  $\Lambda_2$  (Lanne et al. (2010)).

Thus, under the assumptions of orthogonality and state invariant instantaneous effects, the structural shocks are uniquely determined by the transformation  $\varepsilon_t = B^{-1}U_t$ . Thus any further restrictions induced by theoretical

models become over-identifying and testable. Under the normality assumption, the likelihood ratio test is suitable for the purpose. Degrees of freedom of the asymptotic  $\chi^2$  distribution of the test statistic coincide with the number of restrictions being tested.

If there are more than two volatility states, the corresponding covariance matrix decomposition

$$\Sigma_1 = BB', \quad \Sigma_i = B\Lambda_i B', \quad i = 2, \dots, M, \quad (5)$$

with diagonal  $\Lambda_i$  matrices is restrictive. Assumption that the decomposition exists imposes restrictions on the covariance matrices which can be tested. Hence, if there are three or more states with covariance matrices  $\Sigma_1, \dots, \Sigma_M$ , the invariance of the initial effects of the shocks across states can be checked by a likelihood ratio test. According to the null hypothesis being tested, the covariance matrices have representations as in (5). The degrees of freedom for the asymptotic  $\chi^2$  is  $0.5MK(K+1) - K^2 - K(M-1)$ . In words, the number of elements in  $B$  and diagonal elements of  $M-1$   $\Lambda_i, i = 2 \dots M$  matrices are subtracted from the number of distinct elements in the  $M$  covariance matrices (Lanne et al. (2010)).

It is worth pointing out that the requirement of having distinct relative variances is necessary for an exact identification of all shocks. The  $B$  is (locally) unique, if for each pair of equal diagonal elements, say, in  $\Lambda_2 = \text{diag}(\lambda_{21}, \dots, \lambda_{2K})$ , there is a corresponding pair of distinct diagonal elements in one of the other  $\Lambda_i = \text{diag}(\lambda_{i1}, \dots, \lambda_{iK})$  (Lanne et al. (2010)). For instance, if  $\lambda_{2k} = \lambda_{2l}$  then  $\lambda_{ik} \neq \lambda_{il}$  for  $i = 3, \dots, M$  must exist. An important advantage of the approach adopted in this paper is that the equality of  $\lambda_{mi}$ s can be checked with Wald and likelihood ratio tests. Wald tests do not require the full optimization of the model under the alternative that is advantageous in the current setup. However, it may happen that the standard errors of the  $\Lambda_m$ s are poorly estimated. In this situation, LR tests may be useful. Further discussion of tests for two- and three-state MS models can be found in Lanne et al. (2010) and Herwartz and Lütkepohl (2011).

Since I assume the normality of the residuals conditional on the states, the likelihood function can be set up and the model is estimated by maximum likelihood (ML). The concentrated likelihood function and detailed discussion of the related estimation problems can be found in Herwartz and Lütkepohl (2011). In the present paper the expectation maximization (EM) algorithm of Herwartz and Lütkepohl (2011) is adopted and updated to allow for changes in the intercept. The likelihood function is nonlinear, therefore numerical optimization is used. For estimation purposes, I bound diagonal elements of  $\Lambda_i, i = 2, \dots, M$  matrices away from zero. The optimization runs for a set of starting values to reduce the possibility of getting stuck in a local optimum.

## 3.2. Bootstrapping confidence bands

In the MS models, bootstrapping confidence bands for impulse responses may be problematic; therefore, discussion of the procedure deployed in the present paper is useful. Herwartz and Lütkepohl (2011) propose a fixed design wild bootstrap for constructing confidence intervals for impulse responses. They suggest constructing bootstrap samples conditional on estimated state probabilities and the ML estimates. For the current model, I take into account the changes in the intercepts when constructing the bootstrapped series. One of the ways to do this is to use a weighted average of the intercept for each  $t$ , with the weights being the estimated state probabilities. Thus for the current model, the bootstrapped series can be represented as:

$$Y_t^* = \mu_t + \hat{A}_1 Y_{t-1} + \cdots + \hat{A}_p Y_{t-p} + U_t^*, \quad (6)$$

where  $\mu_t = (\hat{\xi}_t \hat{\nu}_{st})'$  and  $\hat{\xi}_t = [\hat{\xi}_{1t}, \dots, \hat{\xi}_{Mt}]$  is a  $1 \times M$  vector of estimated state probabilities for period  $t$ ,  $\hat{\nu}_{st}$  is a  $M \times K$  matrix of estimated state dependent intercepts,  $U_t^* = \eta_t \hat{U}_t$  and  $\eta_t$  is a random variable that has Rademacher distribution (takes values 1 and  $-1$  with probability 0.5).

Note that I do not bootstrap a history of the hidden regimes but rather take it as given following Herwartz and Lütkepohl (2011). I bootstrap parameter estimates  $\theta^*$  of  $\theta = \text{vec}[\nu_{st}, A_1, \dots, A_p]$  and  $B^*$  of  $B$ , conditional on the initially estimated transition probabilities. Therefore the weights for the intercept in the bootstrap loop do not change.

Note that Herwartz and Lütkepohl (2011) also condition on the estimated  $\Lambda_i, i = 2, \dots, M$ , matrices. I relax this assumption and estimate the weighting matrices in the bootstrap step. In order to eliminate any potential interchanges of columns of the  $B$  matrix one needs to impose an ordering of the diagonal elements of  $\Lambda_i, i = 2, \dots, M$ , for unrestricted models. However, no additional ordering of the relative variances (diagonal elements of  $\Lambda_i$ ) is required if just-identifying restrictions on the  $B$  or  $\Xi_\infty$  are imposed.

Apart from this, in each iteration of the bootstrap, I check if signs of the diagonal elements of the  $B^*$  are consistent with the signs of the diagonal elements of the initial estimate  $\hat{B}$ . This is done to avoid interchanges in signs of the  $B$  and to reduce confidence bands as discussed by Lütkepohl (2012). In general, to fix the sign, one should choose elements in the  $\hat{B}$  with the lowest standard errors and carry over the signs to the bootstrap loop. For instance, I fix the elements on the main diagonal of the  $B$  to be positive for productivity-hours data. Then at each bootstrap step, if an element on the main diagonal of the  $B^*$  is negative, the relevant column of the  $B^*$  is multiplied by  $-1$ . Note

that this procedure is simply a device for reducing confidence bands for impulse responses.

It should be emphasized that computing the bootstrapped impulse responses in this way requires a nonlinear optimization of the log-likelihood as in the maximization step of the EM algorithm and is computationally demanding. I use ML estimates of  $\hat{\theta}$  as starting values in each bootstrap replication. In the empirical analysis, I consider 90% percentile confidence intervals based on 1000 replications.

## 4. Empirical analysis

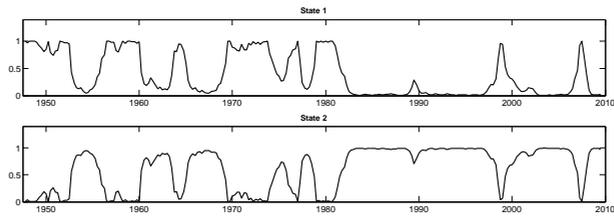
### 4.1. Statistical Analysis

For the purpose of validating restrictions, MS models with two and three states are estimated. In the current section I will focus primarily on the analysis of two state models. As will become clear further down, main arguments regarding the identification will be valid independent of the number of states. Where any differences are detected, they will be discussed. Detailed results for three state models are available as an appendix upon request.

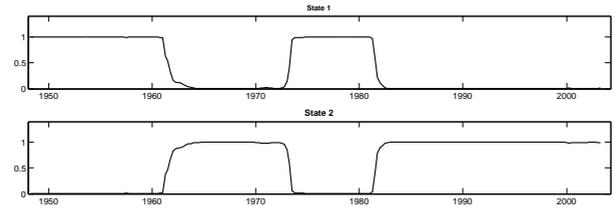
In Table 2, the range of estimated two-state models together with the corresponding values of the log-likelihood, the Akaike Information Criterion (AIC) and the Schwarz Criterion (SC), are presented. In the current study, models with different restrictions for each of the datasets are compared. First, according to the information criteria, the models with MS are preferred to the standard VAR models. As can be seen the values of the AIC and SC are reduced when the identifying restrictions are imposed. For the investment specific technology shock model, the SC favors the most restrictive model, whereas the AIC supports a set of restrictions that identify non-technology shock leaving investment specific and neutral technology shocks unidentified.

In addition to the model selection criteria, it is useful to look at the smoothed state probabilities. These are shown in Figure 1 for the two-state models. The corresponding state covariance matrices for two-state models are given in Table 3. The figures show that volatility changes are present during the sample period. From Table 3 it becomes clear that the States 1 and 2 of the MSIH(2) models can be interpreted as high and low volatility states, respectively. The variances of all of the variables are significantly lower in State 2 relative to State 1.

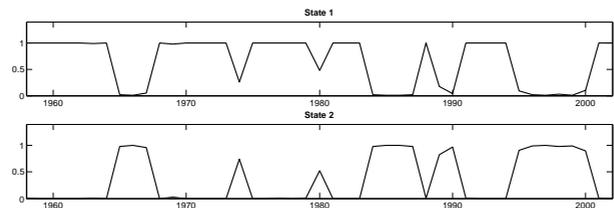
Periods of high volatility can be associated with the periods of economic downturns in the sample period for Models 1 and 2 (Figure 1(a)). The esti-



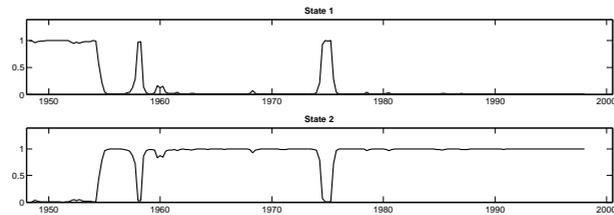
(a) Models 1 and 2



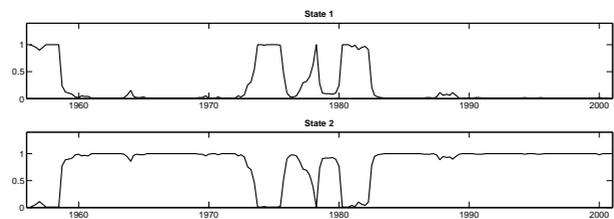
(b) Model 3



(c) Model 4



(d) Model 5



(e) Model 6

Figure 1: Smoothed state probabilities of MSIH(2) models

Table 2: Comparison of MSIH(2) Models

Model	Restrictions	$\text{Log}L_t$	AIC	SC
Model 1	VAR without MS	1696.55	-3351.10	-3277.06
	Unrestricted	1729.88	-3403.76	-3305.49
	$C_{12}(1) = 0$	1729.42	<b>-3404.85</b>	<b>-3310.09</b>
Model 2	$C_{21}(1) = 0$	1728.31	-3402.61	-3307.86
Model 3	VAR without MS	1442.10	-2842.20	-2770.55
	Unrestricted	1489.28	-2922.55	-2827.53
	$C_{12}(1) = 0$	1488.94	<b>-2923.88</b>	<b>-2832.25</b>
Model 4	VAR without MS	239.17	-452.34	-428.29
	Unrestricted	258.84	-477.69	-440.68
	$C_{12}(1) = 0$	258.32	<b>-478.64</b>	<b>-443.48</b>
Model 5	VAR without MS	1969.01	-3848.02	-3698.70
	Unrestricted	2026.71	-3941.42	-3756.96
	$C_{12}(1) = C_{21}(1) = 0$	2025.08	<b>-3942.16</b>	<b>-3762.98</b>
Model 6	VAR without MS	-523.23	1136.46	1280.89
	Unrestricted	-492.99	1097.98	1276.16
	$C_{12}(1) = 0$	-494.30	1098.60	1273.91
	$C_{13}(1) = C_{23}(1) = 0$	-493.85	<b>1095.72</b>	1268.83
	$C(1)$ lower tr.	-497.04	1100.08	<b>1268.71</b>

Note:  $L_T$  - likelihood function,  $AIC = -2 \log L_T + 2 \times \text{no of free parameters}$ ,  $SC = -2 \log L_T + \log T \times \text{no of free parameters}$ .

Table 3: Estimated State Covariance Matrices of MSIH(2) Models

Model	$\Sigma_1 \times 10^{-3}$	$\Sigma_2 \times 10^{-3}$
Models 1 and 2	$\begin{bmatrix} 0.124 & & \\ 0.017 & 0.093 & \\ & & \end{bmatrix}$	$\begin{bmatrix} 0.033 & & \\ -0.007 & 0.026 & \\ & & \end{bmatrix}$
Model 3	$\begin{bmatrix} 0.264 & & \\ -0.063 & 0.107 & \\ & & \end{bmatrix}$	$\begin{bmatrix} 0.045 & & \\ -0.008 & 0.030 & \\ & & \end{bmatrix}$
Model 4	$\begin{bmatrix} 0.212 & & \\ 0.085 & 0.501 & \\ & & \end{bmatrix}$	$\begin{bmatrix} 0.016 & & \\ -0.001 & 0.023 & \\ & & \end{bmatrix}$
Model 5	$\begin{bmatrix} 0.114 & & & \\ 0.006 & 0.122 & & \\ 0.029 & 0.128 & 0.539 & \\ & & & \end{bmatrix}$	$\begin{bmatrix} 0.067 & & & \\ 0.001 & 0.042 & & \\ 0.001 & 0.011 & 0.049 & \\ & & & \end{bmatrix}$
Model 6	$\begin{bmatrix} 996.3 & & & \\ -202.3 & 827.1 & & \\ 522.3 & 287.1 & 1323.1 & \\ & & & \end{bmatrix}$	$\begin{bmatrix} 573.0 & & & \\ -36.8 & 158.4 & & \\ 7.3 & -7.5 & 247.5 & \\ & & & \end{bmatrix}$

mated state probabilities reveal the great moderation phenomena that started at the beginning of the 80s and lasted until the late 90s. A similar picture can

be seen in Figure 1(c), where a measure of TFP is substituted for productivity. Estimated smoothed probabilities of the other datasets may not have a clear economic interpretation. For instance, both 3 variable models have a long duration of the low volatility state with a high volatility state being in place at the beginning and in the middle of the samples.

I intend to use the MS structure for identification purposes; therefore, the main question of interest is whether assumptions needed for local identification are satisfied. Recall from the Section 3 that to obtain a statistical identification of the shocks for a two-state model, it is enough to check whether the associated relative variances of unrestricted models are sufficiently different from each other. The estimates of  $\lambda_{2i}$ s together with the estimated standard errors for a range of MSIH(2) models are shown in Table 4. The standard errors indicate that the estimation precision is quite good for the two variable models and reasonable for the three variable models. Hence, I anticipate that the estimates are statistically different.

Table 4: Estimates of structural parameters of unrestricted MSIH(2) Models

Data	$\hat{\lambda}_{21}$	std.dev	$\hat{\lambda}_{22}$	std.dev	$\hat{\lambda}_{23}$	std.dev
Models 1 and 2	0.181	0.060	0.396	0.099		
Model 3	0.169	0.038	0.309	0.081		
Model 4	0.025	0.032	0.122	0.213		
Model 5	0.090	0.074	0.437	0.373	0.589	0.943
Model 6	0.138	0.068	0.244	0.162	1.021	0.573

Note: Standard errors are obtained from the inverse of the outer product of numerical first order derivatives.

Recall that  $B$  is locally identified in the two-state model (apart from changes in the sign and permutation of its columns) if each pair of the diagonal elements of the  $\Lambda_2$  matrix is distinct. For the two-dimensional system I thus have to check the equality of one pair of the diagonal elements  $\lambda_{21}$  and  $\lambda_{22}$ . For the three variable systems, three pairwise equalities must be checked. In the related literature Wald and likelihood ratio (LR) tests are used in the context (see, for example, Lanne et al. (2010) and Herwartz and Lütkepohl (2011)). Given that some of the standard errors of the  $\Lambda_2$  elements shown in Table 4 are relatively high, the Wald test may perform poorly. Therefore I use computationally more demanding LR tests. The results are presented in Table 5. For the two variable models, the null hypotheses are rejected by the tests at a 5% significance level. For the three variable models the situation is somewhat different. For the capital tax augmented data, one pairwise equality cannot be rejected at a high level with  $p = 0.418$ . The remaining tests produce  $p$  values

at around 0.2. Recall that there are only two restrictions to test using this data, and therefore the very high  $p$  value is not a big problem. Test results for the last specification are better, with the highest  $p$  value being 0.189 and the others below 10%. The results for the three-variable models may be caused by an imprecisely estimated  $\Lambda_2$  rather than by the true equality of its diagonal elements, as the required covariance matrix decomposition always exists for two states. The relatively short duration of the second state may have influenced the estimation precision of  $\Lambda_2$ .

Table 5: Test for Equality of  $\lambda_{ij}$ s for unrestricted MSIH(2) Models

Data	$H_0 : \lambda_{21} = \lambda_{22}$		$H_0 : \lambda_{22} = \lambda_{23}$		$H_0 : \lambda_{21} = \lambda_{23}$	
	LR	$p$	LR	$p$	LR	$p$
Models 1 and 2	6.275	0.012				
Model 3	4.221	0.039				
Model 4	5.955	0.014				
Model 5	1.625	0.202	1.623	0.203	0.656	0.418
Model 6	1.720	0.189	2.810	0.093	12.267	0.001

Note:  $LR = 2(\log L_T - \log L_T^r)$ , where  $L_T^r$  denotes the maximum likelihood under  $H_0$  and  $L_T$  denotes the maximum likelihood for the model under  $H_1$ . Here under  $H_1$  are unrestricted MSIH(2) Models.

Hence, based on the LR tests, there is strong evidence in favor of a unique  $B$  for the two variable models, as well as enough evidence for three variable models. This means that I have achieved a statistical identification of the two-state models. The shocks obtained are unique but they are not labeled economically. With this identification in hand the economic restrictions on  $\Xi_\infty$  become overidentifying. The main question is whether the data supports the economically meaningful technology shocks identified by Gali (1999), Francis and Ramey (2005), Chang and Hong (2006) and Canova et al. (2010).

The usual LR tests are applicable to perform the testing of the restrictions. A small Monte Carlo experiment shows, that the probability to reject a true null hypothesis is 7%, showing the test has reasonable power. The outcomes of the LR tests are shown in Table 6. The LR test for the datasets support the lower-triangular  $\Xi_\infty$  matrix at 5% level for all the models, with the exception of investment-specific technology data. One can reject the identification scheme of investment specific and neutral technology shocks at 5% level. To understand the sources of the rejection, additional models with identified neutral and non-technology shocks are estimated. The separate identification of these two shocks is supported by the data. Therefore imposing both restrictions simultaneously leads to the rejection of the lower triangular  $\Xi_\infty$  matrix. The non-permanent technology shock identified in Francis and Ramey (2005)

also seems to have less support from the data with  $p = 0.07$ . Clearly, the remaining ways of identifying technology shocks are consistent with the properties of the data.

Table 6: LR Tests of Restrictions for MSIH(2) Models

Model	Restriction under $H_0$	DF	LR	$p$ -value
Model 1	$C_{12}(1) = 0$	1	0.92	0.337
Model 2	$C_{21}(1) = 0$	1	3.14	0.07
Model 3	$C_{12}(1) = 0$	1	0.67	0.41
Model 4	$C_{12}(1) = 0$	1	1.05	0.30
Model 5	$C_{12}(1) = C_{21}(1) = 0$	2	3.26	0.19
	$C_{12}(1) = 0$	1	2.26	0.13
Model 6	$C_{13}(1) = C_{23}(1) = 0$	2	1.74	0.42
	$C(1)$ lower tr.	3	8.10	0.04

Note:  $LR = 2(\log L_T - \log L_T^r)$ , where  $L_T^r$  denotes the maximum likelihood under  $H_0$  and  $L_T$  denotes the maximum likelihood for the model under  $H_1$ . Here under  $H_1$  are unrestricted MSIH(2) Models.

The following differences in the testing outcomes should be mentioned for the three-state models. State invariant instantaneous responses are not supported for Models 4, 5, and 6. For Model 4, the restriction is rejected at 5% level in favor of the fully unrestricted three-state model. The identifying restrictions are strongly rejected for both three-variable models. However, this result is driven by the rejection of the state invariant  $B$  for Model 5 and Model 6. For Model 5, the identifying restriction itself cannot be rejected at 5% level for the alternative of state invariant  $B$ . The identification of investment specific technology shocks for Model 6 is rejected at a high level, while the identification of neutral technology shocks is not.

In Table 7 the relative variances of the structural shocks for the restricted models are presented. Model 6 is omitted as the joint identification of investment specific and neutral technology shocks is not supported by the data. The relative variances associated with technology shocks are  $\hat{\lambda}_{21}$ s for all models. If  $\lambda_{21} > \lambda_{22}$ , then the volatility of the shock associated with  $\lambda_{21}$  is higher than the volatility of the shock associated with  $\lambda_{22}$ . The technology shocks as in Galí (1999) and Francis and Ramey (2005) are more volatile in the low volatility states than the non-technology shocks. As the low volatility state can be associated with good times in the economy, it is reasonable that non-technology shocks (say, demand shocks) exhibit low volatility. There is more variation in the technology shocks, as technological development is an ongoing process. Plausibly, the technological innovations are implemented in good times when more resources are available, hence giving rise to adjustments in

productivity rather than employment. Moreover, productivity is more volatile than hours in both states (see Table 3).

Table 7: Estimates of structural parameters of restricted MSIH(2) Models

Data	$\hat{\lambda}_{21}$	std.dev	$\hat{\lambda}_{22}$	std.dev	$\hat{\lambda}_{23}$	std.dev
Model 1	0.381	0.098	0.189	0.044		
Model 2	0.357	0.088	0.208	0.045		
Model 3	0.179	0.038	0.301	0.085		
Model 4	9.726	17.14	64.81	63.86		
Model 5	3.043	1.601	11.59	4.580	1.562	0.911

*Note: Standard errors are obtained from the inverse of the outer product of numerical first order derivatives.*

The TFP shocks exhibit a lower increase in variance in the high volatility state than non-TFP shocks. Hence, when the economy is in turbulent times, the work force – rather than TFP – reacts. In the high volatility state the technology shocks after controlling for capital tax have higher volatility than capital tax shocks but lower than non-technology shocks. The work force exhibits more pressure in bad times relative to technology, supporting the result for the TFP shocks.

The outcome of the testing can be briefly summarized as follows: (1) the identification of permanent technology shocks as in Gali (1999), non-permanent technology shocks, and permanent real wage shocks as in Francis and Ramey (2005) is supported for the models with two and three Markov states; (2) permanent TFP shocks as in Chang and Hong (2006) and permanent technology shocks after controlling for capital tax (Francis and Ramey (2005)) are supported in two-state models; (3) disentangling investment-specific and neutral technology shocks as in Fisher (2002) and Canova et al. (2010) is not supported by the data independent of the number of states. However, a neutral technology shock can be identified in the system. With these results, impulse response analysis is performed.

## 4.2. Impulse Response Analysis

Given that the majority of the identification schemes were supported by the data, the impulse response (IR) analysis may be performed for the supported identification. The impulse responses for the variables that enter in first differences are accumulated. Some of the impulse responses fall outside the respective 90% bootstrap confidence bands. This feature has also been observed in

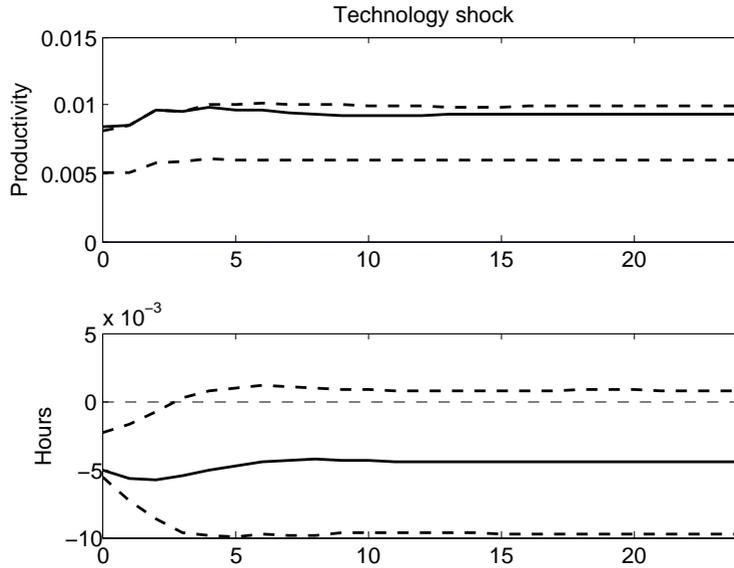
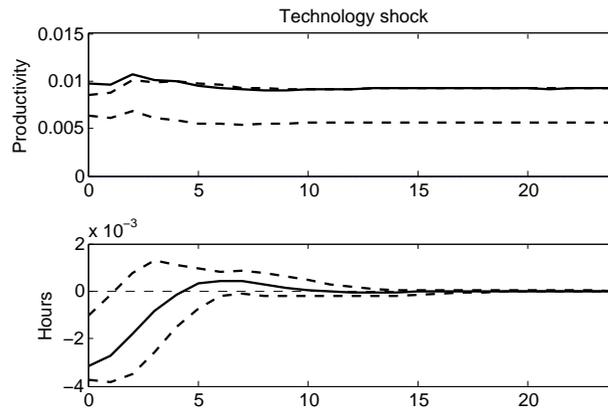


Figure 2: Responses to a positive technology shock, Model 1

some other studies and is not uncommon in the VAR literature. In the current study it might be due to a complex optimization step in the bootstrap cycle.

In Figure 2, the responses to the technology shock identified as in Gali (1999) are shown. The responses are consistent with the previous findings in the literature ((Gali (1999), Christiano et al. (2003), Francis and Ramey (2005) and others): productivity improves significantly, while hours are negative on impact; they then rise but remain negative. It should be noted that the upper confidence band starting from around 4-th quarter is above 0. This feature is also common to the results in the related literature.

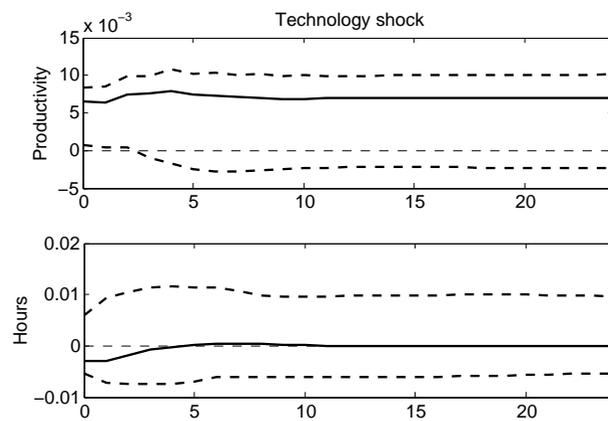
In Figure 3, the responses to non-permanent technology and real wage shock identified in Francis and Ramey (2005) are shown. The dynamics is similar to the permanent technology shock: productivity (real wages) increase while hours worked drop on impact. The lowest bound of the 90% confidence interval for the response of hours to wage shock is actually above zero at horizon five and later. Put differently the response of hours is positive at that particular confidence level. However, the lower bound is so close to zero that for a wider confidence interval the response would become insignificant. Figure 3(c) shows the reaction of productivity and hours to technology shocks after controlling for capital tax. The responses do not change much with respect to Figure 3(a). The response of productivity is positive on impact and of hours worked still insignificant. Hence, there is no full proof evidence in



(a) Response to non-permanent technology shock, Model 2



(b) Response to real wage shock, Model 3



(c) Responses to a positive technology shock after controlling for capital tax, Model 5

Figure 3: Responses to shocks identified as in Francis and Ramey (2005)

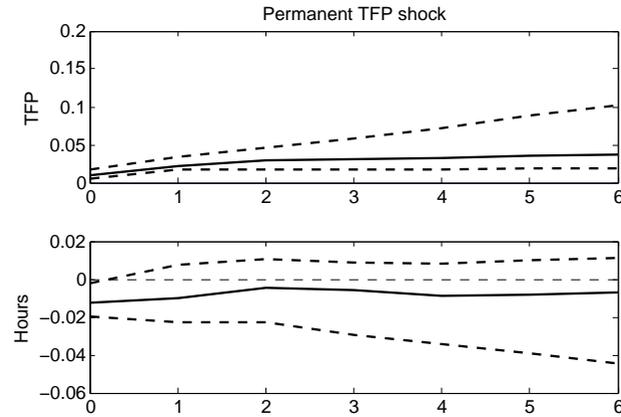


Figure 4: Responses to a positive TFP shock, Model 4

favor of positive and significant reaction of hours to these types of technology shocks.

Figure 4 shows the impulse responses to the TFP shock as in Chang and Hong (2006). On impact, the TFP is positive and hours negative. Then both start rising. Hours do not become positive in the horizon of 6 years; moreover, the reaction is insignificant on nearly all the response horizon. This contrasts findings in Chang and Hong (2006). This should not be surprising, given that the lower confidence band is only slightly above zero in this study (Chang and Hong (2006), Figure 1).

The last specification studied disentangled neutral and investment-specific technology shocks. Recall that full identification was not supported by the data, but an identified neutral technology shock was supported for two- and three-state models. Therefore it may be useful to study the impulse responses of the system with an identified neutral technology shock and try to identify the investment-specific technology shocks. The responses are shown in Figure 5. The investment-specific technology shock is identified as the only shock that positively and significantly increases real price of investment on impact. Unfortunately, the confidence bands for the impulse responses are quite wide for the model. One can see that in the limit the response of hours worked is positive, but that it is insignificant for both types of technology shocks.

Given the variety of studied impulse responses, there is no clear-cut strong evidence in favor of a positive and significant reaction of hours to different technology shocks, although it is plausible for some models.

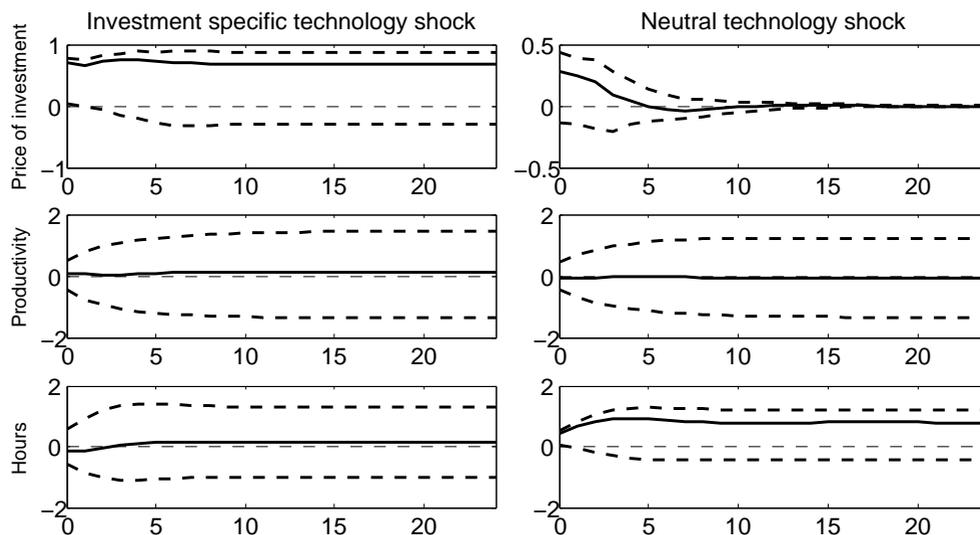


Figure 5: Responses to an investment specific and neutral technology shocks, Model 6

## 5. Conclusions

In the present paper I reconsider the effect of technology shocks on productivity and hours worked. I use Markov switching VAR instead of a standard VAR and assume that the intercept and variance-covariance matrices change over time. The reason for doing this is that proposals has been made to use heteroskedasticity in order to complement and test just-identifying economic restrictions. Identification via heteroskedasticity is particularly useful in the current analysis as there are several ways to identify technology shocks discussed in the literature.

Different identification schemes with long-run restrictions are used by Gali (1999), Francis and Ramey (2005), Chang and Hong (2006), Fischer (2002) and Canova et al. (2010). The studies listed above propose studying permanent and non-permanent technology shocks, permanent real wage shocks, permanent TFP shocks as well as to disentangle investment-specific and neutral technology shocks. In the conventional framework potentially competing restrictions are just-identified and hence not testable. In contrast, the present setup of the econometric model allows for the extraction of additional information from the data and to test just-identifying long run restrictions.

The results of the testing procedure show that the identification of perma-

nent technology shocks as in Gali (1999), non-permanent technology shocks and permanent real wage shocks as in Francis and Ramey (2005) is supported by the data. Further, permanent TFP shocks as in Chang and Hong (2006) and permanent technology shocks after controlling for capital tax (Francis and Ramey (2005)) are supported in two state models. Finally, disentangling investment-specific and neutral technology shocks as in Fisher (2002) and Canova et al. (2010) is not supported by the data independent of the number of states. However, a neutral technology shock can be identified in the system.

Finally, given the variety of impulse responses studied, I conclude that there is no strong evidence in favor of a positive reaction of hours to technology shocks. A positive and significant reaction is plausible only for a real wage shocks and investment specific-neutral technology shocks tandem. The latter result is achieved even though the original identification by Fisher (2002) and Canova et al. (2010) is rejected by the data. However, a better way of computing impulse responses would be useful for more precise inference.

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